Solutions to JEE Main Home Practice Test - 2 | JEE - 2024

PHYSICS

SECTION-1

1.(B) The electric filed at the centre of a charged semicircular wire is given by $E=2\left[\frac{1}{4\pi\varepsilon_0}\frac{Q}{\pi r^2}\right]$ (r = radius of semicircular wire)

$$\Rightarrow \quad E_{net}=2\left[\frac{1}{4\pi\varepsilon_0}\right]\left[\frac{Q}{\pi r^2}-\frac{Q}{\pi R^2}\right]=\frac{2Q}{4\pi\varepsilon_0\pi}\left[\frac{1}{r^2}-\frac{1}{R^2}\right]$$

$$=\frac{2\times0.7\times10^{-9}\times9\times10^{9}}{3.14}\left[\frac{1}{(0.1)^2}-\frac{1}{(0.2)^2}\right]=\frac{2\times0.7\times9}{3.14}\left[10^2-\frac{10^2}{4}\right]$$

$$=\frac{2\times0.7\times9\times10^2(0.75)}{3.14}=3.009\times10^2\approx301\,V/m$$

2.(C) $T_1=2\pi\sqrt{\frac{l}{g}}$ or $\frac{4\pi^2 l}{T_1^2}=g$

$T_2=2\pi\sqrt{\frac{l}{g+a}}$ or $\frac{4\pi^2 l}{T_2^2}=g+a$

$T_3=2\pi\sqrt{\frac{l}{g-a}}$ or $\frac{4\pi^2 l}{T_3^2}=g-a$

$\therefore \quad \frac{4\pi^2 l}{T_2^2}+\frac{4\pi^2 l}{T_3^2}=2g$ where $g=\frac{4\pi^2 l}{T_1^2}$

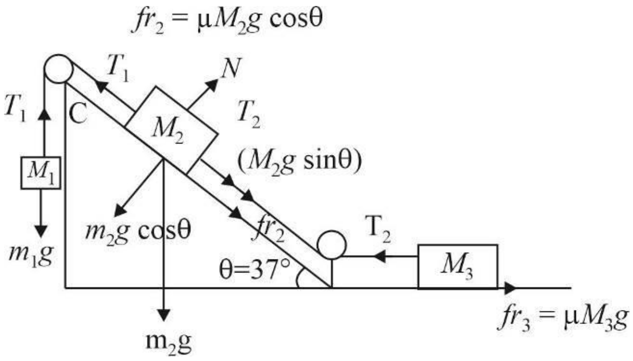
Solving we get : $T_1=\frac{\sqrt{2}T_2T_3}{\sqrt{T_2^2+T_3^2}}$

3.(C) Draw FBD for all the three blocks and apply Newton' second law for all the three blocks. Then, solve the expression to get answer.

$f_r=$ frictional force

For block M_1, $m_1 g=T_1$ ……. (i)

For block M_2, [As $fr_2=\mu m_2 g\cos\theta$]



$T_1=T_2+M_2 g\sin(37°)+\mu M_2 g\cos37°$

$M_1 g = T_2 + M_2 g(\sin 37° + \mu \cos 37°)$ ……. (ii)

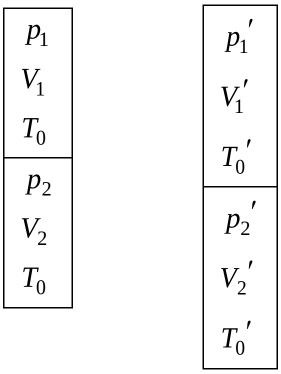
For lock $M_3,\ T_2 = \mu M_3 g\ [fr_3 = \mu M_3 g]$ ……. (iii)

From Eqs. (ii) and (iii), we get

$\Rightarrow \quad M_1 g = \mu M_3 g + M_2 g(\sin 37° + \mu \cos 37°)$

$\Rightarrow \quad M_1 = 0.25 \times 4 + 4\left(\frac{3}{5} + 0.25 \times \frac{4}{5}\right) = 1 + \frac{16}{5} = \frac{21}{5} = 4.2kg$

4.(D) The situation is shown in figure.



If m be the mass of piston and s its cross-sectional area, then $p_1 + \frac{mg}{s} = p_2$ and $p_1' + \frac{mg}{s} = p_2'$

From these equations, we have $p_2 - p_1 = p_2' - p_1'$ …….. (i)

For an ideal gas $p_1 = \frac{RT_0}{V_1},\ p_2 = \frac{RT_0}{V_2}$

$p_1' = \frac{RT}{V_1'},\ p_2' = \frac{RT}{V_2'}$

Substituting these values in Eq. (i), we get

$RT_0\left(\frac{1}{V_2} - \frac{1}{V_1}\right) = RT\left(\frac{1}{V_2'} - \frac{1}{V_1'}\right)$ or $\frac{T_0}{V_1}\left(\frac{V_1}{V_2} - 1\right) = \frac{T}{V_1'}\left(\frac{V_1'}{V_2'} - 1\right)$

or $T = T_0\left(\frac{\eta - 1}{\eta' - 1}\right)\frac{V_1'}{V_1}$ …….. (ii)

where $\frac{V_1}{V_2} = \eta$ and $\frac{V_1'}{V_2'} = \eta'$

From figure, $V_1' + V_2' = V_1 + V_2$ or $V_1'\left[1 + \frac{V_2'}{V_1'}\right] = V_1\left[1 + \frac{V_2}{V_1}\right]$

or $\frac{V_1'}{V_1} = \frac{[1 + (1/\eta)]}{[1 + (1/\eta')]} = \frac{\eta'(\eta + 1)}{\eta(\eta' + 1)}$ …….. (iii)

From Eqs. (ii) and (iii), we get $T = T_0\left[\left(\frac{\eta - 1}{\eta' - 1}\right)\left\{\frac{\eta'(\eta + 1)}{\eta(\eta' + 1)}\right\}\right]$

Here, $T_0 = 300K,\ \eta = 4$ and $\eta' = 3$

Substituting these values in Eqs. we get

$T = 300\left[\left(\frac{4 - 1}{3 - 1}\right)\left\{\frac{3(4 + 1)}{4(3 + 1)}\right\}\right] = 300\left[\frac{3}{2}\left\{\frac{3}{4} \times \frac{5}{4}\right\}\right] = 421.9K$

5.(A) $F_r = -\frac{du}{dr} = -Kr$

For circular Motion

$|F_r| = K_r = \frac{mV^2}{r}$

$\Rightarrow \quad kr^2 = mV^2$ . . . . (i)

Bohr's Quantization $mvr = \frac{nh}{2\pi}$ . . . . .(ii)

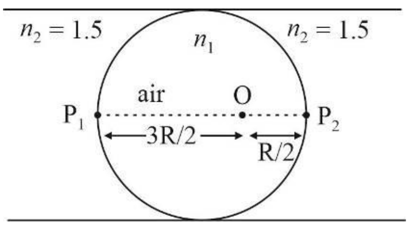
From (i) and (ii) $\frac{m^2V^2}{m} = kr^2$

$\Rightarrow \quad r = \left(\frac{h^2}{4\pi^2 mk}\right)^{1/4} n^{1/2} \Rightarrow r \propto \sqrt{n}$

$K.E. = \frac{1}{2}mV^2 \; PE = \frac{1}{2}kr^2 + \frac{kr^2}{2} = kr^2 \propto n$

6.(C) For the refraction through curved surface $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

Refraction through left surface $\frac{3/2}{v_1} - \frac{1}{\left(\frac{-3}{2}R\right)} = \frac{(3/2-1)}{-R}$



$\frac{3}{2v_1} = \frac{(-3-4)}{6R}$

$\frac{3}{2v_1} = \frac{-7}{6R}$

$v_1 = \frac{-9}{7}R = -1.28R \quad \Rightarrow \quad 1.28\,R$ from p_1

Refraction through right surface

$\frac{3/2}{v_2} - \frac{1}{(-R/2)} = \frac{(3/2-1)}{-R}$

$\frac{3}{2v_2} + \frac{2}{R} = \frac{-1}{2R}$

$\frac{3}{2v_2} = \frac{-1}{2R} - \frac{2}{R}$

$\frac{3}{2v_2} = \frac{(-1-4)}{2R}$

$\frac{3}{2v_2} = \frac{-5}{2R}$

$v_2 = \frac{-3R}{5} = -0.6R \quad \Rightarrow$ So separation between the two images

$= R - (0.28 + 0.6)R = R - [0.88]R = 0.12R$

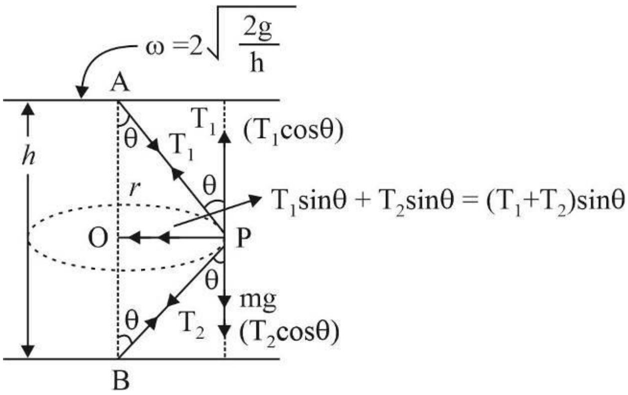
7.(B) From the figure it is clear that object P is tied with identical strings, so $\angle OAP = \angle PBO = \theta$ Here, $(T_1 + T_2)\sin\theta$ will provide required centripetal force.

So, $(T_1 + T_2)\sin\theta = mr\omega^2$ ……. (i)

and $T_1 \cos\theta = mg + T_2 \cos\theta \quad \cos\theta(T_1 - T_2) = mg$ ……. (ii)

By Eqs. (i) and (ii), we get $\frac{(T_1 + T_2)\sin\theta}{(T_1 - T_2)\cos\theta} = \frac{r\omega^2}{g}$

$\frac{(T_1 + T_2)}{(T_1 - T_2)}\tan\theta = \frac{\omega^2}{g}\left[\tan\theta \times \frac{h}{2}\right] \left[\because \tan\theta = \frac{r}{(h/2)}\right]$



$\frac{(T_1 + T_2)}{(T_1 - T_2)} = \frac{\omega^2 h}{2g} = \left(4 \times \frac{2g}{h}\right) \times \frac{h}{2g} \quad \left(\text{As, } \omega = 2\sqrt{2g/h}\right)$

$(T_1 + T_2) = 4(T_1 - T_2) \quad \Rightarrow \quad T_1 + T_2 = 4T_1 - 4T_2$

$3T_1 = 5T_2 \quad \Rightarrow \quad \frac{T_1}{T_2} = \frac{5}{3}$

8.(C) $\lambda_d = \frac{h}{mV}$ (De'Broglie wavelength of electron)

$\lambda_p = \frac{C}{v}$ (photon)

So, $\frac{h}{mV} = \frac{C}{v} \times 10^{-3}$ (Given condition)

Solving, $V = 1.46 \times 10^6 m/s$

9.(A) de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mk}} \left(As\, \lambda = \frac{h}{p}\right)$

$0.5 \times 10^{-9} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times m_{He} \times \frac{3}{2}KT}}$ [k = kinetic energy]

$0.5 \times 10^{-9} = \frac{6.6 \times 10^{-34}}{\sqrt{4 \times 1.67 \times 10^{-27} \times 3 \times 1.38 \times 10^{-23} \times T}}$

Solving $T \approx 6.6K$

10.(B) Magnetic field at the centre of orbit $B = \frac{\mu_0 I}{2r}$ ……. (i)

$$I = \frac{e}{T} = \frac{e}{\frac{2\pi r}{v}} = \frac{ev}{2\pi r} \quad \text{……. (ii)}$$

From Eqs. (i) and (ii), we get $B = \frac{\mu_0}{2r}\left[\frac{ev}{4\pi r}\right] = \frac{\mu_0 ev}{4\pi r^2}$

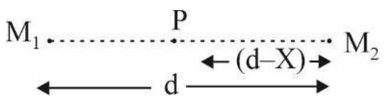
Dividing both sides by L

$$\Rightarrow \quad \frac{B}{L} = \frac{\frac{\mu_0}{4\pi}\frac{ev}{r^2}}{mvr} = \frac{\mu_0 e}{4\pi m r^3}\left(\text{As } L = mvr\right)$$

So, $B = \left(\frac{\mu_0 e}{4\pi m r^3} L\right)$

11.(A) Let at point P the net gravitational field is zero.

$$\Rightarrow \quad \frac{GM_1}{X^2} = \frac{GM_2}{(d-X)^2}$$



[M_1 = mass of 1st body, M_2 = mass of 2nd body, G_1 = gravitational constant]

$$\Rightarrow \quad \frac{M_1}{M_2} = \left(\frac{X}{d-X}\right)^2 \quad \Rightarrow \quad \frac{d-X}{X} = \sqrt{\frac{M_2}{M_1}} \quad \Rightarrow \quad \frac{d}{X} - 1 = \sqrt{\frac{M_2}{M_1}}$$

$$\Rightarrow \quad \frac{d}{X} = 1 + \sqrt{\frac{M_2}{M_1}} \quad \Rightarrow \quad \frac{d}{X} = \frac{\sqrt{M_1} + \sqrt{M_2}}{\sqrt{M_1}} \quad \Rightarrow \quad X = \frac{\left(\sqrt{M_1}\right)}{\sqrt{M_1} + \sqrt{M_2}}$$

At X net gravitational field is zero. So, net gravitational potential at P

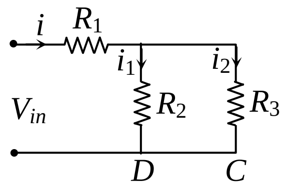
[V = gravitational potential at point P]

$$V = \frac{-GM_1}{X} + \left(\frac{-GM_2}{d-X}\right) = -G\left[\frac{M_1\left(\sqrt{M_1} + \sqrt{M_2}\right)}{d\sqrt{M_1}} + \frac{M_2\left(\sqrt{M_1} + \sqrt{M_2}\right)}{d\sqrt{M_2}}\right]$$

$$= \frac{-G}{d}\left[\left(\sqrt{M_1} + \sqrt{M_2}\right)\left(\sqrt{M_1} + \sqrt{M_2}\right)\right] = \frac{-G}{d}\left(\sqrt{M_1} + \sqrt{M_2}\right)^2$$

$$V = \frac{-G}{d}\left(M_1 + M_2 + 2\sqrt{M_1 M_2}\right)$$

12.(C) As the voltage in R_2 and R_3 is same therefore, according to,



$$H = \frac{V^2}{R} \cdot t \ , \quad R_2 = R_3$$

Also the energy in all resistance is same.

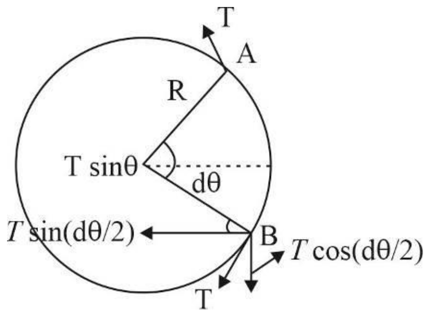
$$\therefore \quad i^2 R_1 t = i_1^2 R_2 t$$

Using $i_1 = \frac{R_3}{R_2 + R_3} i = \frac{R_3}{R_3 + R_3} i = \frac{1}{2} i$

Thus, $i^2 R_1 t = \frac{i^2}{4} R_2 t$ or $R_1 = \frac{R_2}{4}$

13.(A) Let us consider a small Arc AB = dS

The components of T, $T\cos\left[\frac{(d\,\theta)}{2}\right]$ will cancel each other.



$2T\sin\left(\frac{d\theta}{2}\right)$ will provide the required centripetal force.

So, $2T\sin\left(\frac{d\theta}{2}\right)=(dm)R\omega^2$ (dm = mass of small arc)

For small angle, $\sin\left(\frac{d\theta}{2}\right)\approx\left(\frac{d\theta}{2}\right)$ [As if $\theta <<< \tan\theta=\sin\theta=\theta$]

$\Rightarrow \quad 2T\left(\frac{d\theta}{2}\right)=dm\,R\omega^2 \quad [\rho=\text{density}, \omega=\text{angular velocity}]$

$Td\theta=dm\,R\omega^2$; $Td\theta=(dS\times\rho)R\omega^2\times A$

$T=\frac{dS}{d\theta}\times\rho R\omega^2\times A\left(R=\frac{dS}{d\theta}\right)$ (dS : Arc length)

$T=\rho R^2\omega^2\times A[T=\text{tension ....(i)}]$

From Young's modules, $Y=\frac{T/A}{\Delta l/l}$ .... (ii)

$[\Delta l=\text{change in length}]$

Let the length of ring is l.

$\Rightarrow \quad l=2\pi r \Rightarrow \Delta l=2\pi\Delta R \quad \Rightarrow \frac{\Delta l}{l}=\frac{\Delta R}{R}$ .... (iii)

From Eqs. (ii) and (iii), we get $\frac{\Delta R}{R}=\frac{T}{AY}$ [$\Delta R=$ change in reading]

$\Rightarrow \quad \Delta R=\frac{RT}{AY}$ .... (iv)

From Eqs. (i) and (iv), we get

$\Rightarrow \quad \Delta R=\frac{R}{AY}\left[\rho R^2\omega^2\times A\right]\Rightarrow \Delta R=\frac{\rho R^3\omega^2}{Y}$

14.(B) For the pitot tube by Bernaulli's theorem and Pascal's law $\frac{1}{2}\rho v^2=\Delta p=\rho_0 g\Delta h$

So, speed of the gas $v=\sqrt{\frac{2\rho_0 g\Delta h}{\rho}}$ .... (i)

Now, rate of gas flow $(Q), Q=v\times s$ .... (ii)

From (i) and (ii), we get $Q=\sqrt{\frac{2\rho_0 g\Delta h}{\rho}}\cdot s$

15.(A) $\frac{mgl}{2}+mgl=\frac{1}{2}\left(\frac{4}{3}ml^2\right)\omega^2,\ \omega=\frac{3}{2}\sqrt{\frac{g}{l}}$

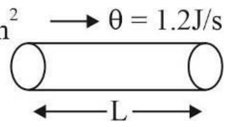
16.(B) A represents R and B represents to X_L.

So, $AB=RX_L=(\Omega)^2$ (unit wise)

$$=\left[\frac{V}{A}\right]^2=\left[\frac{W}{q\times A}\right]^2=\left(\frac{F\times S\times t}{q\times E}\right)$$

$$[AB]=\left(\frac{(MLT^{-1})\times(L)\times(T)}{[A]^2[T^2]}\right)^2=\left[ML^2T^{-3}A^{-2}\right]^2=\left[M^2L^4T^{-6}A^{-4}\right]$$

17.(D) If power P = 1.2W

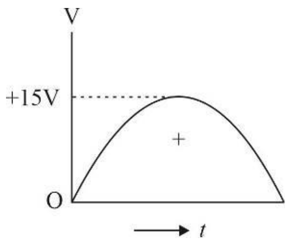


$K = \frac{400W}{mK}, \Delta T = 10°C = 10K$

Rote of heat flow $\frac{\Delta Q}{\Delta t} = \frac{KA\Delta T}{L}$

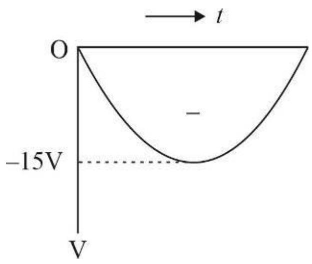
$1.2 = \frac{400 \times 3 \times 10^{-4} \times 10}{L} \Rightarrow L = \frac{12 \times 10^3 \times 10^{-4} \times 10}{1.2} \Rightarrow L = 1m$

18.(A) For positive half,



D_1 will be forward and conduct current and D_2 will be revered biased and it will be not conduct.

To negative half.



D_2 will conduct and D_1 will not conduct.

19.(C) Given $\frac{di_1}{dt} = \frac{di_2}{dt}$

For an inductor $V = L\frac{di}{dt}$

$\therefore \quad \frac{V_2}{V_1} = \frac{I_2}{I_1} = \frac{1}{4}$

As instantaneous powers are equal $V_1 i_1 = V_2 i_2 \quad \Rightarrow \quad \frac{i_1}{i_2} = \frac{V_2}{V_1} = \frac{1}{4}$

$\therefore \quad \frac{W_1}{W_2} = \frac{L_i i_1^2}{L_2 i_2^2} = (4)\left(\frac{1}{4}\right)^2 = \frac{1}{4} \quad \Rightarrow \quad \frac{W_2}{W_1} = 4$

20.(D) $\vec{R}_1 = \vec{F}_1 + \vec{F}_2,\ \vec{R}_2 = \vec{F}_1 - \vec{F}_2$

If α be the angle between $\vec{R}_1$ and $\vec{R}_2$ then $\cos\alpha = \frac{\vec{R}_1.\vec{R}_2}{R_1 R_2} = \frac{F_1^2 - F_2^2}{F_1^2 + F_2^2}$ but $F_1 = F_2$

$\Rightarrow \quad \cos\alpha = 0$ and $\alpha = 90°$

SECTION - 2

1.(24) $8 = 0 + a\left(2 - \frac{1}{2}\right)$ ...(1)

$S_5 = 0 + a\left(5 - \frac{1}{2}\right)$ ...(2)

Dividing equation (2) by (1)

We get, $S_5 = 24\,\text{m}$.

2.(10) $|\Delta\vec{V}| = |\vec{V}_F - \vec{V}_i| = \sqrt{10^2 + 10^2 - 2\times10\times10\cos60^\circ} = \sqrt{100} m/s = 10\ m/s$

3.(8) $V = 5 + 4x^2$ [potential along x-axis]

$E = \frac{-dV}{dx} = -8x$ [Electric field along x-axis]

$E(x=-1) = 8\frac{V}{m}$

So, force along x-axis given by

$\Rightarrow \quad F = qE = 1\times8 = +8N$

4.(15) Tension in the rope = ?

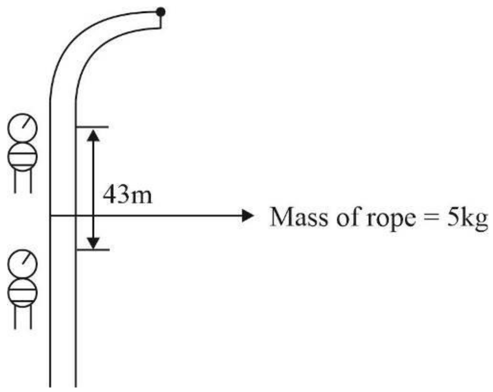
$\text{Speed} = \frac{\text{Distance}}{\text{Time}}, v = \frac{420m}{1.4s} = 300 m/s$

Speed of wave on a string $v = \sqrt{T/\mu}$

[T = tension, μ = mass per unit length]

$300 = \sqrt{\frac{T}{\left(\frac{5}{420}\right)}} \quad \Rightarrow \quad (300\times300) = \frac{T\times420}{5}$

$T = \frac{300\times300\times5}{420} = \frac{15}{14}kN \quad \therefore \quad n = 15$



5.(5) Problem solving strategy Fist apply the condition for isothermal compression and then for adiabatic expansion. For isothermal compression,

$\Rightarrow \quad p_2 = 2p_1$

Isothermal pV = constant

adiabatic $\rightarrow pV^\gamma =$ constant, pV = constant

$\Rightarrow \quad p_1V_1 = p_2V_2,\ p_1V_1 = 2p_1V_2 \Rightarrow V_2/2$

For adiabatic expansion, $pV^\gamma =$ constant

$\Rightarrow \quad p_2V_2^\gamma = p_3V_1^\gamma \quad \Rightarrow \quad 2p_1\left(\frac{V_1}{2}\right)^\gamma = p_3(V_1)^\gamma \qquad \left[\text{As } V_2 = \frac{V_1}{2}\right]$

$\Rightarrow \quad p_3 = 2p_1\left(\frac{1}{2}\right)^\gamma = 2p_1\left(\frac{1}{2}\right)^{7/5}$

$= 2^{1-\frac{7}{5}}p_1 = 2^{-2/5}p_1 \quad \Rightarrow \quad p_3 = \frac{p_1}{2^{2/5}} = 0.76p_1$

6.(3) $\vec{r}$ = position vector of particle $= 2\cos(\omega t)\hat{i} + 3\sin(\omega t)\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = -2\omega\sin(\omega t)\hat{i} + 3\omega\cos(\omega t)\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = -2\omega^2\cos(\omega t)\hat{i} - 3\omega^2\sin(\omega t)\hat{j} = -\omega^2\left[2\cos(\omega t)\hat{i} + 3\sin(\omega t)\hat{j}\right]$

$\vec{a} = -\omega^2\vec{r}$ $\quad\quad \vec{F} = m\vec{a} = -m\omega^2\vec{r}$

$\Rightarrow$ Torque = $|\vec{r}\times\vec{F}| = |r||F|\sin\theta = r\times\left(-m\omega^2 r\right)\sin\pi\left[As\ \theta = 180^\circ\right] = \text{zero}$

7.(4) $V_0 = 283V$

$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{283}{2}\sqrt{2}V$

$X_L = \omega L = 320\times\frac{1}{40}\Omega = 8\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{320\times 1000\times 10^{-6}}\Omega = \frac{1000}{320}\Omega = \frac{25}{8}\Omega$

$X = X_L - X_C = 8 - \frac{25}{8} = \frac{39}{8}\Omega \approx 5\Omega$

$Z = \sqrt{R^2 + X^2} = 5\sqrt{2}\Omega = 5\times 1.414\Omega = 7.\Omega$

$I_{rms} = \frac{V_{rms}}{Z} = \frac{283\sqrt{2}}{2\times 5\sqrt{2}}A = 28.3A$

$\tan\phi = \frac{X}{R} = \frac{5}{5} = 1 \Rightarrow \phi = 45^\circ$

8.(40) Heat needed to bring 50 gm water from $40^\circ C$ to $0^\circ C$

$Q_1 = 50\times 4.2\times 40 = 8400J$

Out of this some heat is provided by 20gm ice.

$Q_2 = 20\times 2.1\times 20 = 840$ J

$Q = Q_1 - Q_2 = 7560$

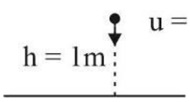
If mass 'm' of ice is melted, then $7560 = m\times 334 + m\times 2.1\times 20$ gives $m \cong 20\ gm$.

So, $\quad m(\text{total}) = 40gm$

9.(90) Coefficient of restitution $e = \frac{\sqrt{2gh'}}{\sqrt{2gh}}$

(h' = height rises when through with velocity $2m/s$, h = height rises when dropped from $1m$)

Here, $\quad h' = \frac{3}{4}h$ and $h = 1m$



$e = \sqrt{\frac{h'}{h}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Speed at the time touching the ground $v_1^2 = u^2 + 2gh = 4 + 2\times 10\times 1$

$v_1 = \sqrt{24}m/s$

So, $e = \frac{v_2}{v_1}$ $\quad$ [v_2 = the speed which the ball will rise] $\frac{\sqrt{3}}{2} = \frac{v_2}{\sqrt{24}}$

$v_2 = \frac{\sqrt{3\times 24}}{2} = \sqrt{18}m/s$

Height up to which the ball will rise $\quad v^2 = u^2 - 2gh \quad\Rightarrow\quad 0 = v_2^2 - 2gh'$

$v_2^2 = 2gh' \quad\Rightarrow\quad 18 = 2\times 10\times h' \quad\Rightarrow\quad h' = 0.9\,m = 90\,cm$

10.(25) When corrective lens is used then eye can see the object at infinity. Power of eye lens in this situation is p_∞

$u=\infty \text{ and } v=2cm=0.02m$

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$P_\infty=\frac{1}{0.02}-\frac{1}{\infty}=50$$

$$P_\infty=50+0$$

$$P_\infty=50D$$

If $P=$ Power of eye at near point when corrective lens is used

$$P=P_\infty+P_a=50+4=54D$$

Let near point in this situation is x_n

$$u=-x_n m$$

$$v=+2cm=0.02m$$

$$\frac{1}{f}=54$$

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$\frac{1}{0.02}+\frac{1}{x_n}=54$ (all distance are in m)

$$50+\frac{1}{x_n}=54$$

$$x_n=\frac{1}{4}m=0.25m=25cm$$

CHEMISTRY

SECTION - 1

1.(A) Neptunium (Np) and plutonium (Pu) show maximum number of oxidation states starting from +3 to +7.

2.(B) $K_b = 10^{-5} \quad \Rightarrow \quad pK_b = 5 \quad \Rightarrow \quad \log 2 = 0.301$

$0.02M\ NH_4Cl$

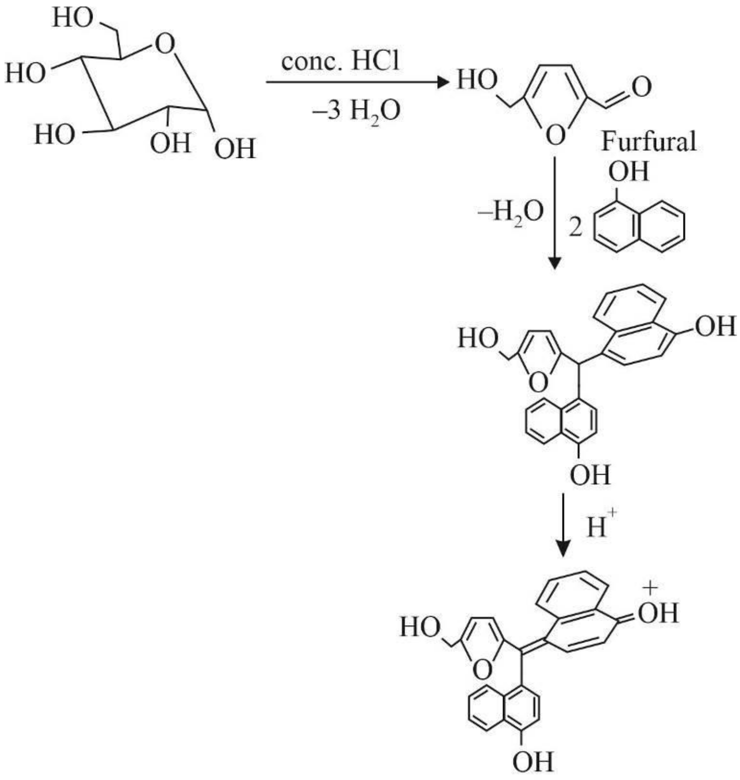
$\underset{\text{Acidic salt}}{NH_4Cl} + H_2O \rightleftharpoons NH_4OH + HCl$

$pH = 7 - \frac{1}{2}(pK_b + \log C) = 7 - \frac{1}{2}\left(5 + \log 2 \times 10^{-2}\right) = 7 - \frac{1}{2}(3.301) = 7 - 1.6505$

$pH = 5.3495$

3.(C) Cr_2O_3 reacts with acid and alkali both where as Sc_2O_3 reacts with acid only.

4.(A) Molisch's test is a test for carbohydrates larger than tetroses. In molisch's test, the carbohydrate undergoes dehydration upon addition of conc. HCl or H_2SO_4. This molecule undergoes condensation with α-naphthol present in reagent, resulting in the formation of purple or reddish-purple coloured complex



Reddish-purple colour complex.

5.(C) $K_1 = 2.5 \times 10^{-4}\ dm^3\ mol^{-1}\ s^{-1}\ T_1 = 327°C \Rightarrow 600K$

$K_2 = 1.0\ dm^3\ mol^{-1}\ s^{-1}\ T_2 = 527°C \Rightarrow 800K$

$E_A \Rightarrow$ in kJ / mole

Given $R = 8.314\ J / K\text{-mole}$

From $\log\frac{K_2}{K_1} = \frac{Ea}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$$\log\frac{1}{2.5}\times 10^{-4} = \frac{Ea}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\log\frac{1}{2.5\times 10^{-4}} = \frac{Ea}{2.303\times 8.314}\left(\frac{1}{600} - \frac{1}{800}\right)$$

$E = 165.54\,kJ$

6.(B) Benzaldehyde can reduce Tollen's Reagent but not Fehling solution

7.(B) CFSE

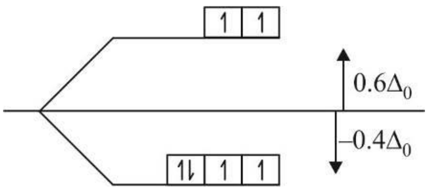
Of $\underset{\text{Octahedral Complex}}{\left[Fe(H_2O)_6\right]}Cl_2$

H_2O weak field Ligand

i.e. do not pair up the unpaired electron.

$\left[Fe(H_2O)_6\right]^{+2}$

$Fe^{+2}\ 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^6$.



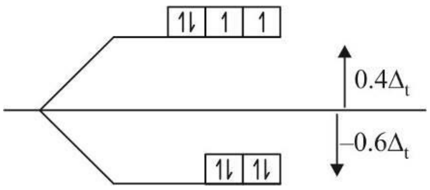
CFSE $\Rightarrow -4\times 0.4\Delta_0 + 2\times 0.6\Delta_0$

$= -1.6\Delta_0 + 1.2\Delta_0$

$= -0.4\Delta_0$

$K_2[NiCl_4]$

$\underset{Ni^{+2}}{[NiCl_4]^{-2}}$ $\underset{sp^3}{\text{Tetrahedral Complex}}$



$Cl^- \Rightarrow$ weak field Ligand do not pair up

$Ni^{+2}\quad 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^8$

$CFSE = -0.6\times 4\Delta_t + 0.4\times 4\Delta_t = -2.4\Delta_t + 1.6\Delta_t$

$CFSE = -0.8\Delta_t$

8.(C) Given $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

$\Delta H = -57.2$ kJ/mol

$K_c = 1.7\times 10^{16}$

(A) From $K\downarrow = A\,e^{-\frac{\Delta H}{RT\uparrow}}$ as T increases, K decreases

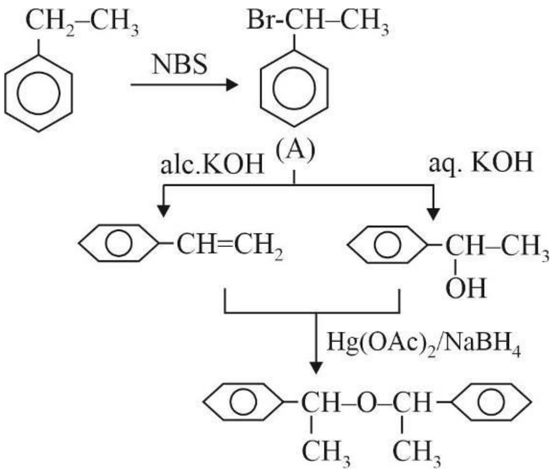
(B) Equilibrium constant is not a affected by change in volume.

(C) Although K_c is large but it doesn't mean reaction go for completion.

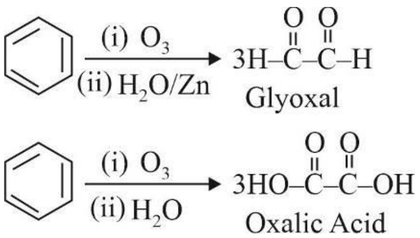
(D) Equilibrium will shift in forward direction as pressure increases.

9.(D) In SO_4^{2-} ion, sulphur can decrease its oxidation state only so it acts as only oxidising agent.

10.(B)

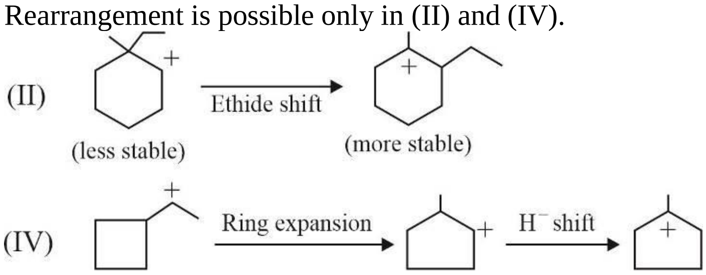


11.(A)



12.(C) Since, this compound have no carbon, CN^- ion is not formed and presence of nitrogen can't be detected.

13.(B) Rearrangement is possible only in (II) and (IV).



14.(A) The wavelength of absorption for complex depending on value of $\Delta_0 = E = \frac{hc}{\lambda}$.

15.(B) **(A)** sp^3 **(B)** dsp^2 **(C)** sp^3d^2 **(D)** sp^3d^2

16.(C) All polysaccharides are non reducing sugars this is due to acetal glycosidic linkage present between monosaccharide units.

17.(B) $$\underset{(X)}{2KMnO_4} \xrightarrow{\Delta} \underset{(Y)}{K_2MnO_4} + MnO_2 + \frac{1}{2}O_2$$

$$MnO_2 + 2Cl^- + 4H^+ \longrightarrow Mn^{+2} + 2H_2O + Cl_2$$

18.(B) $$Cd(OH)_2 \rightleftharpoons Cd^{+2} + 2OH^-$$

$$K_{sp} = [Cd^{+2}][OH^-]^2$$

solubility $s = [Cd^{+2}] = \frac{K_{sp}}{[OH^-]^2}$;

pH of the buffer = 12 ; pOH = 2, $[OH] = 10^{-2}$; $K_{sp} = 4s^3 = 4(1.84 \times 10^{-5})^3$

$$[Cd^{+2}]_f = \frac{4[1.84 \times 10^{-5}]^3}{(10^{-4})} = \frac{4 \times 6.23 \times 10^{-15}}{10^{-4}} = 2.49 \times 10^{-10} M.$$

19.(C) $FeSO_4 \cdot 7H_2O$

(pale green salt)

$Fe^{2+} \xrightarrow{\text{standing in Air}} Fe^{3+}$

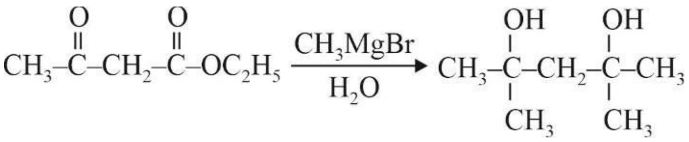
$Fe^{3+} + 3OH^- \longrightarrow \underset{\text{Brown}}{Fe(OH)_3}$

$Fe^{2+} + H_2S \xrightarrow{2OH^-} \underset{\text{Black ppt.}}{FeS} + 2H_2O$

$5Fe^{2+} + MnO_4^- + 8H^+ \longrightarrow Mn^{2+} + 5Fe^{3+} + 4H_2O$

20.(B) If $\Delta G^\circ < O$ then $K_{eql} > 1$

SECTION - 2

1.(3) 

2.(750)

	NaOH	+ HCl	$\longrightarrow$ NaCl	+ H_2O
millimoles	= 250 × 0.5	500 × 1	0	0
	= 125	500	125	125

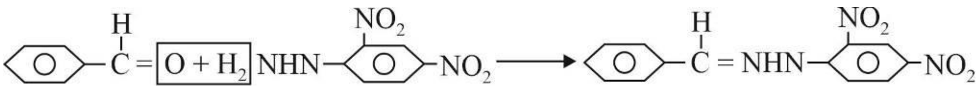
No. of millimoles of NaCl formed = 125

So, no. of moles = 0.125

No. of molecules = $0.125 \times 6 \times 10^{23} = 0.75 \times 10^{23} = 750 \times 10^{20}$

3.(25) Energy produced by eating 160 gm of glucose = $\frac{2800}{180} \times 160 = 2488.8$

Maximum distance travelled by person = $\frac{2488.8}{100} = 24.88\text{km} = 25\text{ km}$

4.(4) 

5.(46) $k = \frac{2.303}{60} \log \frac{100}{100-25}$

$k = \frac{2.303}{60} \log \frac{100}{75}$

$k = \frac{2.303}{60}(\log 4 - \log 3)$

$k = \frac{2.303}{60}(0.6 - 0.48)$

$k = 0.004606 = 46.06 \times 10^{-4}$

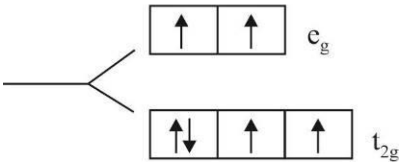
6.(356) $Y_{CCl_4} = \frac{X_{CCl_4}.P^0_{CCl_4}}{X_{CCl_4}.P^0_{CCl_4} + X_{SiCl_4}.P^0_{SiCl_4}}$

$$\frac{\frac{\frac{w}{154}}{w/154 + w/170} \times 115}{\frac{w/154}{w/154 + w/170} \times 115 + \frac{w/170}{w/154 + w/170} \times 230} = 0.356$$

7.(6) This include 2 double bond and 1 lone pair.

8.(2) $[CoF_6]^{3-}$ has Co^{3+}

$Co^{3+} = [Ar]3d^6$



9.(18) $(i - ii) \times 2 =$ equation (iii)

$$\therefore \quad k_p = \left(\frac{k_{p_1}}{k_{p_2}}\right)^2 = \left(\frac{1.8 \times 10^8}{8 \times 10^{-2}}\right)^2 = 0.05 \times 10^{20} = 5 \times 10^{18}$$

10.(12)

S. No.	Electronic Configuration	l	m_ℓ	$(l + m_\ell)$	Electrons
1	$1s^2$	0	0	0	2
2	$2s^2$	0	0	0	2
3	$2p^6$	1	–1	0	2
		0			
		+1			
4	$3s^2$	0	0	0	2
5	$3p^6$	1	–1	0	2
		0			
		+1			
6	$4s^2$	0	0	0	2

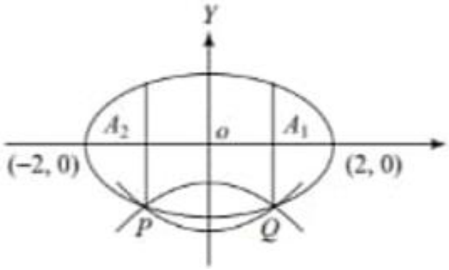
MATHEMATICS

SECTION-1

1.(D) Eccentricity e of the ellipse is given by

$b^2 = a^2(1-e^2) \Rightarrow \quad 1 = 4\left(1-e^2\right) \Rightarrow e = \sqrt{3}/2$.

Focii of the ellipse are $\left(\sqrt{3}, 0\right)$ and $\left(-\sqrt{3}, 0\right)$



Length of a latus rectum of the ellipse is $2\dfrac{b^2}{a} = 1$

Thus, $P\left(x_1, y_1\right) = p\left(-\sqrt{3}, -1/2\right)$ and $Q\left(x_2, y_2\right) = Q\left(\sqrt{3}, -1/2\right)$

Length of the latus rectum PQ of the parabola is $\quad |x_2 - x_1| = 2\sqrt{3} = 4p(say)$

As focus of a parabola is the mid point of the latus rectum, focus of the desired parabola is $\left(0, -1/2\right)$ and hence its vertices are $\left(0, -1/2 \pm p\right)$

i.e. $\left(0, -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\right)$ and $\left(0, -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}\right)$

Thus there are two parabolas having PQ as the latus rectum whose equations are

$$x^2 = 4p\left(y + \frac{1}{2} + \frac{\sqrt{3}}{2}\right) = 2\sqrt{3}y + \sqrt{3} + 3$$

And $x^2 = -4p\left(y + \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\right) = -2\sqrt{3}y - \sqrt{3} + 3$

2.(B) As $1 < 2x^2 - 3 < 2 \qquad \forall x \in \left(\sqrt{2}, \sqrt{\dfrac{5}{2}}\right)$

$\therefore \qquad \displaystyle\int_{\sqrt{2}}^{\sqrt{5/2}} \left[2x^2 - 3\right] dx > 0 \qquad \forall x \in (0, 2)$

$\Rightarrow \qquad g(x) = 0$ should have at least one real root in (0, 2) $\left\{\because g'(x) \neq 0\right\}$

3.(A) Let remaining two observations are x and y

$\therefore \qquad x + y = 14$

Also $\qquad x^2 + y^2 = 100 \quad \Rightarrow \quad |x - y| = 2$

4.(D) $\int_0^x f(t)dt = \int_x^1 t^2 . f(t)dt + \frac{x^{16}}{8} + \frac{x^6}{3} + a$ ……. (i)

For x = 1, $\int_0^1 f(t)dt = 0 + \frac{1}{8} + \frac{1}{3} + a = \frac{11}{24} + a$

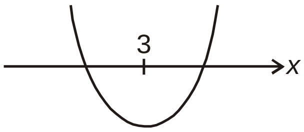
Diff. both sides of (i) w. r. t. x we get;

$f(x) = 0 - x^2 f(x) + 2x^{15} + 2x^5$

$\Rightarrow \quad 2\int_0^1 \frac{x^{15}+x^5}{1+x^2}dx = \frac{11}{24} + a \quad \Rightarrow \quad 2\int_0^1 \left(x^{13} - x^{11} + x^9 - x^7 + x^5\right)dx = \frac{11}{24} + a$

$\Rightarrow \quad 2\left(\frac{1}{14} - \frac{1}{12} + \frac{1}{10} - \frac{1}{8} + \frac{1}{6}\right) = \frac{11}{24} + a \quad \Rightarrow \quad a = -\frac{167}{840}$

5.(C)



D > 0 ,	f(3) < 0
$(a-1)^2 > 4 \times 2 \times 8$	$18 - 3a + 3 + 8 < 0$
$a - 1 > 8$, $a - 1 < -8$	$3a > 29$
$a < -7$, $a > 8$ ...(1)	$a > \frac{29}{3}$ ...(2)

from (1) and (2) $a \in \left(\frac{29}{3}, \infty\right)$

6. (A) Operate $R_1 \to R_1 - \sec x\, R_3$

$$\Rightarrow \quad f(x) = \begin{vmatrix} 0 & 0 & \sec^2 x + \cot x \operatorname{cosec} x - \cos x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$$

$$= \left(\sec^2 x + \cot x \operatorname{cosec} x - \cos x\right)\left(\cos^4 x - \cos^2 x\right)$$

$$= \left(1 + \frac{\cos^3 x}{\sin^2 x} - \cos^3 x\right)\left(\cos^2 x - 1\right)$$

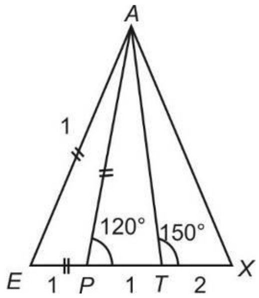
$$= -\sin^2 x \frac{\sin^2 x + \cos^3 x - \cos^3 x \sin^2 x}{\sin^2 x} = -\left(\sin^2 x + \cos^5 x\right)$$

$$\Rightarrow \quad \int_0^{\pi/2} f(x)dx = -\int_0^{\pi/1} \left(\sin^2 x + \cos^5 x\right)dx = -\left(\frac{1}{2}.\frac{\pi}{2} + \frac{4.2}{5.3.1}\right) = -\left(\frac{\pi}{4} + \frac{8}{15}\right)$$

7.(D) If z_1 be the new complex number then $|z_1| = |z| + \sqrt{2} = 2\sqrt{2}$

Also $\frac{z_1}{z} = \frac{|z_1|}{|z|}e^{i3\pi/2} \Rightarrow z_1 = z . 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = 2(1+i)(0-i) = -2i + 2 = 2(1-i)$

8.(D)



(A) $Ax^2 = AE^2 + EX^2 - 2AE.EX\cos\angle AEX = 1 + 6 - 2.1.4.\frac{1}{2} = 13 \quad \therefore \quad AX = \sqrt{13}$

(B) Since $\angle APT$ isosceles $\angle ATP = \angle PAT = 30°$ then $\angle EAT = 90°$

And also $\frac{AT}{\sin 120°} = \frac{AP}{\sin 30°} \Rightarrow AT = \frac{\sqrt{3}}{2}.\frac{1}{1/2} = \sqrt{3}$

(C) Since $EX^2 = AE^2 + AX^2 - 2AE.AX\cos\angle XAE$

$16 = 1 + 13 - 2.1\sqrt{13}.\cos\angle XAE$

$\cos\angle XAE = \frac{-1}{\sqrt{13}}$

9.(B) $f(x) = ax + b$

$f'(x) = a < 0$ (given) $f(x)$ is decreasing function

So $\begin{matrix} f(-1) = 2 \text{(max value)} \Rightarrow & -a + b = 2 \\ f(1) = 0 \quad \text{(min value)} & a + b = 0 \end{matrix} \Bigg\} \begin{matrix} a = -1 \\ b = 1 \end{matrix}$

So $f(x) = -x + 1$

$\Rightarrow \quad f(0) = 1, \; f\left(\frac{1}{4}\right) = \frac{3}{4} \; f(-2) = 3 \; f\left(\frac{1}{3}\right) = \frac{2}{3}$

Now $A = \cos^2\theta + \sin^4\theta$ and we know very well $\frac{3}{4} \le A \le 1$

$\Rightarrow \quad f\left(\frac{1}{4}\right) \le A \le f(0)$

10.(A) We have, $\left(\frac{2 + \sin x}{y + 1}\right)\frac{dy}{dx} = -\cos x, \; y(0) = 1$

$\Rightarrow \quad \int \frac{dy}{y+1} = \int \frac{-\cos x}{2 + \sin x} dx$

$\Rightarrow \quad \log|y + 1| = -\log|2 + \sin x| + c$ ……. (i)

At $x = 0$, $y = 1$

$\Rightarrow \quad \log|2| = -\log|2 + 0| + c \quad \Rightarrow \quad c = 2\log 2$

Putting the value of **(c)** the equation (i) becomes

$\log|y + 1| = -\log|\sin x + 2| + 2\log 2$

$\Rightarrow \quad (y + 1) = \frac{4}{\sin x + 2}$ at $x = \frac{\pi}{2} \Rightarrow y + 1 = \frac{4}{\sin\frac{\pi}{2} + 2} = \frac{4}{3} \Rightarrow y = \frac{4}{3} - 1 = \frac{1}{3} \Rightarrow y\left(\frac{\pi}{2}\right) = \frac{1}{3}$

11.(A) $\dfrac{1}{(1-x)(3-x)}=\dfrac{1}{2}\left[\dfrac{1}{1-x}-\dfrac{1}{3-x}\right]=\dfrac{1}{2}\left[(1-x)^{-1}-(3-x)^{-1}\right]$

$$=\frac{1}{2}\left[(1-x)^{-1}-3^{-1}\left(1-\frac{x}{3}\right)^{-1}\right]=\frac{1}{2}\left[1+x+x^2+x^3+\ldots-\frac{1}{3}\left(1+\frac{x}{3}+\left(\frac{x}{3}\right)^2+\ldots\right)\right]$$

Coefficient of $x^n=\dfrac{1}{2}\left[1-\dfrac{1}{3}.\dfrac{1}{3^n}\right]=\dfrac{3^{n+1}-1}{2.3^{n+1}}$

12.(A) Since the system of equations possess non-trivial solution,

$\begin{vmatrix}\alpha_2 & \alpha_2\\ \beta_1 & \beta_2\end{vmatrix}=0\Rightarrow \alpha_1\beta_2-\alpha_2\beta_1=0$

$\Rightarrow \quad \dfrac{\alpha_1}{\beta_1}=\dfrac{\alpha_2}{\beta_2}=\dfrac{\alpha_1+\alpha_2}{\beta_1+\beta_2}=\left(\dfrac{\alpha_1\alpha_2}{\beta_1\beta_2}\right)^{1/2}$ ……. (1)

Also $\alpha_1+\alpha_2=-\dfrac{b}{a},\alpha_1\alpha_2=\dfrac{c}{a}$ and $\beta_1+\beta_2=-\dfrac{q}{p},\beta_1\beta_2=\dfrac{r}{p}$

$\therefore$ Equation (1) $\Rightarrow \dfrac{-b/a}{-q/p}=\left(\dfrac{c/a}{r/p}\right)^{1/2} \Rightarrow \dfrac{b^2}{q^2}=\dfrac{ac}{pr}.$

13.(C) Coordinates of F_1 are $(ae, 0)$ and of F_2 are $(-ae,0)$

Also $BF_1=BF_2=F_1F_2$

$\Rightarrow \quad a^2e^2+b^2=4a^2e^2$

$\Rightarrow \quad a^2e^2+a^2\left(1-e^2\right)=4a^2e^2 \quad \Rightarrow \quad e=\dfrac{1}{2}\Rightarrow F_1F_2=a$

And the area of the $\Delta BF_1F_2=\dfrac{\sqrt{3}}{4}a^2$

14.(B) We have, $\cot^{-1}\left(\dfrac{n^2-10n+19}{\sqrt{3}}\right)>\dfrac{\pi}{6}$

$\Rightarrow \quad \dfrac{n^2-10n+19}{\sqrt{3}}<\cot\left(\dfrac{\pi}{6}\right) \quad \Rightarrow \quad n^2-10n+25<9$

$\Rightarrow \quad (n-5)^2<3^2 \quad \Rightarrow \quad 2<n<8 \quad \Rightarrow \quad n\in\{3, 4, 5, 6, 7\}$

Clearly, least value of n is 3

15.(C) $\dfrac{dx}{dt}=\cos^2\pi x,$

On differentiating w.r.t. 'x'

$\dfrac{d^2x}{dt^2}=-2\pi\sin(2\pi x)=$ negative

The particle never reaches point, it means $\dfrac{d^2x}{dt^2}=0 \quad \Rightarrow \quad -2\pi\sin 2\pi x=0$

$$\int_0^x\frac{dx}{\sec^2\pi x}=\int_0^t dt$$

$\dfrac{\tan\pi x}{\pi}=t \quad \Rightarrow \quad \tan(\pi x)=\pi t$ as $t\to\infty \quad \Rightarrow \quad x\to\dfrac{1}{2}$

16.(B) $f(x)=\frac{x^2}{2}+\ln x-2\cos x \quad \Rightarrow \quad f'(x)=x+\frac{1}{x}+2\sin x$

We know that $x+\frac{1}{x}\geq 2, \forall\ x>0$ and $2\sin x\leq 2, \forall x\in R$

$\Rightarrow \quad f'(x)>0$ for $x>0$

Hence $f(x)$ is increasing in $(0, \infty)$

When $x<0$, $x+\frac{1}{x}\leq -2$ but $-2\leq 2\sin x\leq 2, \forall x\in R$

$\Rightarrow \quad f'(0)<0$ hence $f(x)$ is decreasing in $(-\infty, 0)$

17(C) Since $g(x)$ is continuous $\forall\ x\in R$, $g(x)$ should be constant

Since $f(x)\in(2, \sqrt{26})$, $a\geq\sqrt{26}, \left(as\left[\frac{f(x)}{\sqrt{26}}\right]=0\forall\ x\in R\right)$

So least integral value of a is 6.

18.(C) 1S, 3A, 1H, 2R, 1N, 1P, 1U when all letters are different.

Corresponding ways = ${}^7C_3.3!\quad {}^7P_3=210$

When two letters are of one kind and other is different.

Corresponding ways = ${}^2C_1.\,{}^6C_1.\frac{3!}{2!}=36$

When all letters are alike, corresponding ways = 1.

Thus total words that can be formed = 210 + 36 + 1, i.e. 247

19.(A) Total formed numbers that begin with a odd digit = ${}^5C_1.\,{}^8P_4=5(8)(7)(6)(5)$

Total formed numbers that end with a odd digit = ${}^5C_1.\,{}^8P_4=(8)(7)(6)(5)$

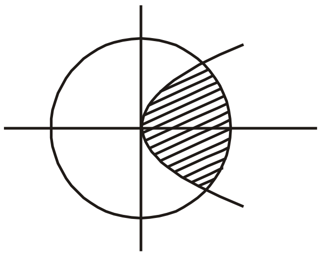
Total formed number that begin with an odd digit and also end with an odd digit

$={}^5C_2.\,2!.\,{}^7P_3=5.(4)(7)(6)(5)$

Thus total formed numbers that begin with an odd digit or end with an odd digits is equal to $5\cdot 7\cdot 6\cdot 60$

Total formed numbers = ${}^9P_5=9.8.7.6.5$ Thus, required probability $=\frac{5}{6}$

20.(A) The point $(-2k, k+1)$ is the interior point of the circle and parabola



So $(2k)^2+(k+1)^2-4<0 \Rightarrow 4k^2+k^2+2k+1-4<0 \Rightarrow 5k^2+2k-3<0$

$(k+1)\left(k-\frac{3}{5}\right)<0\Rightarrow k\in\left(-1, \frac{3}{5}\right)$ .... (1)

Now $(k+1)^2-4(-2k)<0 \Rightarrow k^2+2k+1+8k<0 \Rightarrow k^2+10k+1<0$

$k\in(-5-2\sqrt{6}, -5+2\sqrt{6})$ .... (2)

So from (1) & (2)

$k\in\left(-1,-5+2\sqrt{6}\right)$

SECTION-2

1.(0) Given two lines

$\bar{r} = \bar{a} + t\bar{b}$

$\bar{r} = \bar{c} + s\bar{d}$

x_2 is the foot of the perpendicular drawn from x_1 on to the second line. Again x_3 is the foot of the perpendicular drawn from x_2 on to the first line. This process is repeated so that the point of intersection of two lines is obtained.

2.(11) $AA^T = I$

A is orthogonal matrix.

$A \times A^T = I$

$$\begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 2 + 6 + 3 = 11$

3.(0) $\vec{c} = \frac{\vec{a} - \mu\vec{b}}{4}$ and $\vec{a}.\vec{b} = 0$

Now, $(\vec{b} - \vec{a}).(\vec{b} + \vec{c}) = 0 \Rightarrow (\vec{b} - \vec{a}).\left(\vec{b} + \frac{\vec{a} - \mu\vec{b}}{4}\right) = 0$

$\Rightarrow \quad (4 - \mu)b^2 = a^2 \ \ (\because \mu < 4)$ ……. (i)

Again $4|\vec{b} + \vec{c}|^2 = |\vec{b} - \vec{a}|^2$

$\Rightarrow \quad 4\left|\frac{(4-\mu)\vec{b} + \vec{a}}{4}\right|^2 = |\vec{b} - \vec{a}|^2 \quad \Rightarrow \quad 4\left(\frac{4-\mu}{4}\right)^2 b^2 + \frac{a^2}{4} = b^2 + a^2$

$\Rightarrow \left((4-\mu)^2 - 4\right)b^2 = 3a^2$ ……. (ii)

(i) & (ii) we get $\frac{(4-\mu)^2 - 4}{4 - \mu} = \frac{3}{1} \quad \Rightarrow \quad \mu = 0, 5$

$\Rightarrow \quad \mu = 0$ or 5 but as $\mu < 4$, so, $\mu = 0.$

4.(14) $(a + bx)^{-2} = a^{-2}\left(1 + \frac{b}{a}x\right)^{-2} = \frac{1}{a^2}\left[1 + (-2)\left(\frac{b}{a}x\right) + ...\right] = \frac{1}{a^2} - \frac{2b}{a^3}x + ...$

Also, $(a + bx)^{-2} = \frac{1}{4} - 3x + ...$

$\therefore \quad \frac{1}{a^2} = \frac{1}{4}$ ……. (1)

and $\quad -\frac{2b}{a^3} = -3$ ……. (2)

(1) $\Rightarrow \ a^2 = 4 \ \Rightarrow \ a = 2$ and from (2) $b = 12$

5.(2) $2\vec{a}-3\vec{b}+6\vec{c}=\vec{0} \Rightarrow 2\vec{a}-3\vec{b}=-6\vec{c} \Rightarrow \left|2\vec{a}-3\vec{b}\right|^2=36\left|\vec{c}\right|^2$

$4a^2+9b^2-12\vec{a}.\vec{b}=36c^2$

$4a^2+9b^2-12.ab\cos\theta=36c^2$

$16b^2+9b^2-12.2b^2\cos\theta=36\frac{1}{4}b^2$

$25-24\cos\theta=9 \Rightarrow \cos\theta=\frac{2}{3}$

6.(3) Side of R are given by

$x=\pm 3, y=\pm 2$

Let equation of E_2 be

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

As it passes through (0, 4) and (3, 2) we get $\frac{16}{b^2}=1 \Rightarrow b^2=16$ and $\frac{9}{a^2}+\frac{4}{b^2}=1=a^2=12$

Eccentricity e of E_2 is given by

$a^2=b^2\left(1-e^2\right)$

$\Rightarrow \quad 12=16\left(1-e^2\right) \Rightarrow e=1/2$

7.(6) $\sum_{n=1}^{6}\operatorname{cosec}\left(\theta+(n-1)\frac{\pi}{4}\right)\operatorname{cosec}\left(\theta+\frac{n\pi}{4}\right)=\sqrt{2}\left[\cot\theta-\cot\left(\theta+\frac{3\pi}{r}\right)\right]$

$\Rightarrow \quad \cot\theta+\tan\theta=4 \quad \Rightarrow \quad \theta=15^\circ \text{ or } 75^\circ \quad \therefore \quad \sin^2\beta-\sin^2\alpha=\frac{\sqrt{3}}{2}$

8.(450) Let the number of $n=x_1, x_2, x_3$.

Since $x_1+x_2+x_3$ is even. That means there are following cases :

(i) x_1, x_2, x_3 all are even

$\rightarrow$ 4. 5. 5 = 100 ways

(ii) x_1 even and x_2, x_3 are odd

$\rightarrow$ 4.5.5. = 100 ways

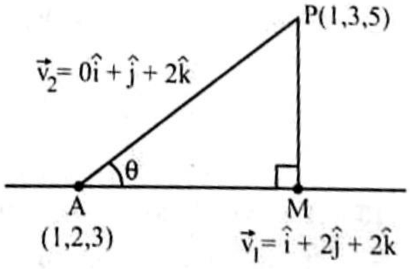
(iii) x_1 odd, x_2 even, x_3 odd

$\rightarrow$ 5.5.5 = 125 ways

(iv) x_1 odd, x_2 even, x_3 odd

$\rightarrow$ 5.5.5. = 125 ways

9.(1) $\therefore \quad PM = |\vec{v}_2| \sin\theta = \sqrt{5}\sin\theta$



As, $\cos\theta = \dfrac{\vec{v}_1.\vec{v}_2}{|\vec{v}_1||\vec{v}_2|} = \dfrac{6}{3\sqrt{5}} = \dfrac{2}{\sqrt{5}}$

$\Rightarrow \quad \sin\theta = \dfrac{1}{5}$

$\therefore \quad PM = |\vec{v}_2| \sin\theta = \sqrt{5}\left(\dfrac{1}{\sqrt{5}}\right) = 1$

10.(3) RH Limit = $\lim\limits_{x\to 2}\{[2-(2+h)]+[(2+h)-2]-(2+h)\} = \lim\limits_{h\to 0}\{[-4h]+[h]-2-h\}$

$\lim\limits_{h\to 0}\{0-1-2+h\} = -3$

LH Limit = $\lim\limits_{h\to 0}\{[2-(2-h)]+[(2-h)-2]-(2-h)\} = \lim\limits_{h\to 0}\{0-1-2+h\} = -3$